2nd Symposium on Analytic Number Theory

8–12 July 2019

Grand Hotel San Michele
Cetraro (CS), Italy

Invited Speakers

A. Booker · J. Brüdern · B. Conrey · C. Dartyge
C. David · R. de la Breteche · S. Drappeau · K. Ford
D. Goldfeld · A. Granville · A. Harper · H. Iwaniec
J. Kaczorowski · J. Keating · D. Koukoulopulos · E. Kowalski
J. Liu · J. Maynard · P. Michel · I. Petrow
L. Pierce · J. Pintz · M. Radziwiłł · Z. Rudnick
N. Snaith · K. Soundararajan · G. Tenenbaum

Organizers

Danilo Bazzanella
Sandro Bettin
Alberto Perelli

The conference is funded by: Politecnico of Torino, University of Genova, Foundation Compositio Mathematica, Istituto Nazionale di Alta Matematica "Francesco Severi" and GNAMPA - INdAM Gruppo Nazionale per l'Analisi Matematica, la probabilità e le loro applicazioni.

8 July 2019

8:30 Registration

9:00 H. Iwaniec, Rutgers University
A zero density estimate for the Dirichlet L-functions

9:50 B. Conrey, American Institue of Mathematics
Main terms in moments

10:30 Coffee Break

11:00 J. Kaczorowski, Adam Mickiewicz University, Poznań
Results on the standard twist of L-functions

11:50 C. David, Concordia University
Moments of cubic Dirichlet twists over function fields

13:00 Lunch

17:30 A. Granville, Université de Montréal
Two sieves revisited

18:20 R. de la Breteche, Université Paris Diderot-Paris 7
Minimizing GCD sums and applications

20:00 Dinner

The morning and afternoon sessions are chaired by D. Fiorilli and C. Elsholtz.

9 July 2019

9:00 K. Soundararajan, *Stanford University*
Integral Factorial Ratios

9:50 J. Brüdern, *Universität Göttingen*
The diophantine properties of spherical Fano threefolds

10:30 Coffee Break

11:00 N. Snaith, *University of Bristol*
Zeros, moments and determinants

11:50 A. Booker, *University of Bristol*
TBA

13:00 Lunch

17:30 M. Radziwiłł, *California Institute of Technology*
The local Fourier uniformity conjecture for multiplicative functions

18:20 S. Drappeau, *Aix-Marseille Université*
Central values of additive twists of L functions via continued fractions

20:00 Dinner

The morning and afternoon sessions are chaired by M. Munsch and A. Walker.

10 July 2019

9:00 J. Maynard, University of Oxford
Primes in arithmetic progressions to large moduli

9:50 D. Koukoulopulos, Université de Montréal
On the Duffin-Schaeffer conjecture

10:30 Coffee Break

11:00 G. Tenenbaum, Université de Lorraine
Sums of digits in different bases

11:50 L. Pierce, Duke University
Burgess methods in higher dimensions

13:00 Lunch

17:00 K. Ford, University of Illinois at Urbana-Champaign
Prime number models, large gaps, prime tuples and the square-root sieve

17:50 Z. Rudnick, Tel Aviv University
The least common multiple of polynomial sequences

18:40 C. Dartyge, Universit de Lorraine
On irregularities of distribution of binary sequences relative to arithmetic progressions

20:00 Dinner

The morning and afternoon sessions are chaired by S. Chow and C. Swaenepoel.

11 July 2019

9:00 D. Goldfeld, Columbia University
A new method to compute the Fourier coefficients of Langlands Eisenstein series

9:50 A. Harper, University of Warwick
Multiplicative chaos in number theory

10:30 Coffee Break

11:00 P. Michel, EPFL Lausanne
Algebraic Twists of GL(3)*-modular forms*

11:50 I. Petrow, EPFL Lausanne
The fourth moment of Dirichlet L-functions along subgroups and the Weyl bound

13:00 Lunch

20:00 Dinner

The morning sessions are chaired by J. Brandes and G. Cherubini.

12 July 2019

9:00 J. Pintz, *Alfréd Rényi Institute of Mathematics*
An approximate formula for Goldbach's problem with applications

9:50 E. Kowalski, *ETH Zürich*
Bilinear forms with exponential sums

10:30 Coffee Break

11:00 J. Liu, *Shandong University*
The disjointness conjecture for skew products

11:50 J. Keating, *University of Bristol*
Arithmetic statistics and mixed moments of elliptic curve L-functions over function fields

13:00 Lunch

20:00 Dinner

The morning session is chaired by A. Zaccagnini.

Abstracts

A. Booker, University of Bristol
TBA

J. Brüdern, Universität Göttingen
The diophantine properties of spherical Fano threefolds

We describe joint work with Blomer, Derenthal and Gagliardi concerning the predictions of Manin and Peyre for the class of varieties in the title line, and beyond.

B. Conrey, American Institue of Mathematics
Main terms in moments

We focus on the arithmetic sources of main terms in the moment conjectures, especially for the Riemann zeta-function.

C. Dartyge, Universit de Lorraine
On irregularities of distribution of binary sequences relative to arithmetic progressions

We introduce some weighted measures to study the distribution in arithmetic progressions, of the short subsequences of binary sequences. We will presents some results for the typical and minimal values of these measures for binary sequences of a given lenght. Next we will give some constructive bounds for these minimal values. This a joint work with Katalin Gyarmati and András Sárközy.

C. David, Concordia University
Moments of cubic Dirichlet twists over function fields

We obtain an asymptotic formula for the mean value of L-functions associated to cubic characters over $F_q[T]$. We solve this problem in the non-Kummer setting when $q \equiv 2 \mod 3$ and in the Kummer case when $q \equiv 1 \mod 3$. The proofs rely on the study of averages of cubic Gauss sums over function fields, which can be understood using the theory of metaplectic Eisenstein series. In the non-Kummer setting we display some explicit cancellation between the main term and the dual term coming from the approximate functional equation of the L?functions. This is joint work with A. Florea and M. Lalin.

R. de la Breteche, Université Paris Diderot-Paris 7
Minimizing GCD sums and applications

We present results related to the minimal values of certain sums of Gál

type, with applications.

S. Drappeau, Aix-Marseille Université
Central values of additive twists of L functions via continued fractions

This talk will be about joint work with S. Bettin (Genova), in which we obtain limiting distributions for averages of continued fractions coefficients of rational numbers, ordered by size. This generalizes work of Baladi and Vallée to situations where the limiting law may not be Gaussian. We will discuss applications, in particular to the distribution of central values $D(1/2, x)$ of the Estermann function, and of modular symbols.

K. Ford, University of Illinois at Urbana-Champaign
Prime number models, large gaps, prime tuples and the square-root sieve

We introduce a new probabilistic model for primes, which we believe is a better predictor for large gaps than the models of Cramer and Granville. We also make strong connections between our model, prime k-tuple counts, large gaps and the "square-root sieve". In particular, our model makes a prediction about large prime gaps that may contradict the models of Cramer and Granville, depending on the tightness of a certain sieve estimate. This is joint work with Bill Banks and Terence Tao.

D. Goldfeld, Columbia University
A new method to compute the Fourier coefficients of Langlands Eisenstein series

We present a new and surprisingly simple method to completely compute the non-constant terms of Langlands Eisenstein series. This is joint work with Michael Woodbury and Stephen D. Miller.

A. Granville, Université de Montréal
Two sieves revisited

We present a new combinatorial approach to the Fundamental Lemma of the sieve, as well as an alternative to Vaughan's identity which can make some consequences of the large sieve easier and stronger.

A. Harper, University of Warwick
Multiplicative chaos in number theory

Multiplicative chaos is the general name for a family of probabilistic objects, which can be thought of as the random measures obtained by taking the exponential of correlated Gaussian random variables. Quite surprisingly, multiplicative chaos turns out to be closely connected with

various problems in analytic number theory, including the value distribution of the Riemann zeta function on the critical line, the moments of character sums, and various model versions of these problems coming from harmonic analysis and probability. I will try to give a gentle introduction to these issues and connections, presenting both results and open problems without assuming too much background knowledge.

H. Iwaniec, Rutgers University
A zero density estimate for the Dirichlet L-functions

I will sketch a proof of an estimate which improves the classical result of Ingham-Montgomery.

J. Kaczorowski, Adam Mickiewicz University, Poznań
Results on the standard twist of L-functions

For an L-function $F(s) = \sum_{n=1}^{\infty} a(n)n^{-s}$, $(\sigma > 1)$, of degree $d > 0$ from the Selberg class, the standard twist is defined as follows

$$F(s, \alpha) = \sum_{n=1}^{\infty} \frac{a(n)}{n^s} e(-\alpha n^{1/d}).$$

Here α is a real parameter, and for a generic real number θ, $e(\theta) = \exp(2\pi i\theta)$ denotes the complex exponential. The latter series converges absolutely for $\sigma > 1$ and admits meromorphic continuation to the whole complex plane. Analytic properties of $F(s, \alpha)$ are of the first importance in the description of the Selberg class and thus deserve a detailed study. The talk is intended to give an overview of them and will end with the latest and jet unpublished ones concerning among others the functional equation, pole structure, and the order of magnitude on verticals. This is joint work with A. Perelli.

J. Keating, University of Bristol
Arithmetic statistics and mixed moments of elliptic curve L-functions over function fields

I will discuss recent calculations relating to arithmetic statistics and mixed moments of elliptic curve L-functions over function fields, in the latter case with applications to rank correlations.

D. Koukoulopulos, Université de Montréal
On the Duffin-Schaeffer conjecture

E. Kowalski, ETH Zürich
Bilinear forms with exponential sums

Many results in analytic number theory depend on estimates for short exponential sums. We will survey some recent estimates of short gen-

eral bilinear forms with coefficients of the form $K(mn)$, where K is a trace function. We will discuss in particular the case where K is a hyper-Kloosterman sum modulo a prime, in which case one can prove non-trivial estimates in ranges similar to the Burgess bound for short character sums. We will also mention some applications to L-functions. This is joint work with Ph. Michel and W. Sawin.

J. Liu, Shandong University
The disjointness conjecture for skew products

The disjointness conjecture of Sarnak states that the Mobius function is disjoint with dynamical systems of zero entropy. In this talk I will describe how to establish this conjecture for a class of skew products. This is joint work with Wen Huang and Ke Wang.

J. Maynard, University of Oxford
Primes in arithmetic progressions to large moduli

We show various results about primes in arithmetic progressions beyond the 'square-root barrier', building on work of Bombieri, Fouvry, Friedlander, Iwaniec, Zhang and others. The proof relies on a combination of sieve methods, the spectral theory of automorphic forms and estimates from algebraic geometry.

P. Michel, EPFL Lausanne
Algebraic Twists of GL(3)*-modular forms*

The subconvexity problem for GL(1)-twists of a fixed GL(3) cusp form (solved by R. Munshi in 2015) is equivalent to establishing that Dirichlet characters χ modulo q do not correlate with the Fourier Whittaker coefficients of the given GL(3) form in the convexity range $q^{3/2}$. In this talk, we will explain how a recent alternative proof of Munshi's theorem -due to R. Holowinsky and P. Nelson- makes it possible to replace the character χ by the trace function of a general ℓ-adic sheaf modulo q (when q is prime). This is joint with with E. Kowalski, Y. Lin and W. Sawin.

I. Petrow, EPFL Lausanne
The fourth moment of Dirichlet L-functions along subgroups and the Weyl bound

L. Pierce, Duke University
Burgess methods in higher dimensions

The Burgess method set a well-known record for bounding short multiplicative character sums in one dimension. This talk will present a

"Burgess method" for character sums in arbitrary dimensions, involving both additive and multiplicative characters, evaluated at appropriate polynomials. This includes joint work with Junyan Xu.

J. Pintz, Alfréd Rényi Institute of Mathematics
An approximate formula for Goldbach's problem with applications

In the lecture we sketch an approximate formula for the contribution of the major arcs in Goldbach's problem as a function of a relatively small number of zeros of Dirichlet L-functions, lying extreme near to the boundary line $\Re s = 1$. This formula plays a crucial role in the proof of the following approximate forms of the Goldbach conjecture. By the use of other ideas and methods we can show, for example, the following approximate forms of the binary Goldbach conjecture.

Concerning the size of the exceptional set in Goldbach's problem we can show:

Theorem 1 (J. P.). The number of even integers up to N which are not representable as the sum of two primes is at most $O(N^{0.72})$.

We can improve the earlier results in the unconditional case of the Linnik-Goldbach problem:

Theorem 2 (J. P. and I.Z. Ruzsa). Every sufficiently large even integer can be written as the sum of two primes and eight powers of 2.

We can give a stronger estimate for the number of Goldbach exceptional numbers in polynomial sequences, than the one proved by Brüdern, Kawada and Perelli. We just mention the result in case if the even integer to be represented as the sum of two primes is the double of a square.

Theorem 3 (A. Perelli and J. P.) Let $c > 4/5$ be fixed. The number of even integers of the form $2n^2$ which are not representable as the sum of two primes is for $n < N$ at most $O(N^c)$.

M. Radziwiłł, California Institute of Technology
The local Fourier uniformity conjecture for multiplicative functions

I will motivate and describe the progress on the so-called "local Fourier uniformity conjecture" and its consequences for our understanding of triple correlations of multiplicative functions, such as the Liouville or divisor function. This is joint work with Matomaki and Tao.

Z. Rudnick, Tel Aviv University
The least common multiple of polynomial sequences

A relatively recent conjecture due to Cilleruelo states that for an irreducible nonlinear polynomial f with integer coefficients of degree $d > 1$, the least common multiple of the sequence $f(1), f(2), ..., f(N)$ has

asymptotic growth $(d-1)N\log(N)$ as N goes to infinity. I will discuss the background and status of this conjecture, and recent work with Sa'ar Zehavi, where we establish a version of this conjecture for almost all shifts of a fixed polynomial, the range of N depending on the range of shifts. Time permitting, I will also discuss a function field version.

N. Snaith, University of Bristol
Zeros, moments and determinants

For 20 years we have known that average values of characteristic polynomials of random unitary matrices provide a good model for moments of the Riemann zeta function. Now we consider mixed moments of characteristic polynomials and their derivatives, calculations which are motivated by questions on the distribution of zeros of the derivative of the Riemann zeta function.

K. Soundararajan, Stanford University
Integral Factorial Ratios

I will describe a new approach to classifying integral factorial ratios, obtaining a direct proof of a result of Bober. These results generalize an observation going back to Chebyshev that $(30n)!n!/((15n)!(10n)!(6n)!)$ is an integer for all natural numbers n. Due to the work of Rodriguez-Villegas and Beukers and Heckman, this problem is closely related to classifying hypergeometric functions with finite monodromy groups, and the result of Bober was originally derived as a consequence of the work of Beukers–Heckman. The new proof is elementary and makes partial progress on other related questions.

G. Tenenbaum, Université de Lorraine
Sums of digits in different bases

Let $s_a(n)$ denote the sum of digits of the expansion of the integer n in base a. We shall describe results from a forthcoming paper with R. La Bretèche and T. Stoll showing that, whenever a and b are multiplicatively independent, the sequence $s_b(n)/s_a(n)$ is everywhere dense in the set of positive real numbers. Explicit estimates for the associated counting functions will also be given.