Abstracts

Italy-France Analytic Number Theory Workshop
DIMA, Univ. Genova

8-10 July 2024

Speakers

Giacomo Cherubini (INdAM)

Monday 8/7, 9:30 - 10:10.

Hyperbolic circle problem in three dimensions

We review the definition and the current status of the hyperbolic circle problem, starting with the two-dimensional case and then moving to 3D. The strongest approach to counting lattice points inside a hyperbolic ball goes by the spectral theory of automorphic forms and gives a main term and an explicit error term with power saving. In the error, both the distribution of the eigenvalues and the size of the eigenfunctions (at a given point) play a role. Our main result, in the three-dimensional case, is that a local average around the center of the hyperbolic ball leads to a better error term than when the center is fixed, as is expected. The main motivation is to compare the saving with the one obtained in two dimensions, proved by Petridis and Risager. Ingredients of the proof are the quantum ergodicity of eigenfunctions and bounds on a spectral sum over the eigenvalues.

Francesco Battistoni (U. Milano Cattolica)

Monday 8/7, 10:20 - 11:00.

The first and second moment for the length of the period of the continued fraction expansion for $\sqrt{d}$

This is a joint work with Loıc Grenié and Giuseppe Molteni.

Given a positive integer d which is not a square, denote by $T(d)$ the length of the period of the continued fraction expansion for $\sqrt{d}$. We prove upper bounds for the first two moments of $T(d)$ by studying the moments of a different function $g(d)$, originally introduced by Hickerson: we detect the asymptotic of the first moment of $g(d)$ and an upper bound for the second moment. The results allow to improve the estimates for the size of the sets of integers d for which $T(d) > \alpha\sqrt{d}$, with α a real parameter. We also report recent progress by Korolev on the asymptotic for the second moment of $g(d)$.

Alessandro Gambini (U. Roma Sapienza)

Monday 8/7, 11:30 - 12:10.

Harmonic sums, Bernoulli numbers and connections with the p-adic zeta function

In 1991 Eswarathasan and Levine proposed a conjecture regarding the distribution of the p-adic valuation of harmonic numbers $H_n = \sum_{k=1}^{n} \frac{1}{k}$. The conjecture asserts that the set J_p of positive integers n for which p divides the numerator of H_n is finite. The complexity of the statement lays in the connection between harmonic numbers and Bernoulli numbers, which led to an expression of the harmonic numbers in terms of the p-adic zeta function. By inverting this formula, it is possible to obtain an expression for the p-adic value of integers, concluding that the p-adic zeta function is non-null for all primes up to 10^{11} and for all non-Wolstenholme primes.

Lucile Devin (U. Littoral Côte d'Opale)

Monday 8/7, 12:20 - 13:00.

Zeros of L-functions and the distribution of Gaussian primes

In this talk, we are interested in the following question: given a prime that can be written as a sum of two squares $p = a^2 + 4b^2$, how does the congruence class of $a > 0$ distribute? This will lead us to study distribution of values of Hecke characters from the point of view of Chebyshev's bias, as well as distribution of zeros of the associated L-functions and in particular vanishing at 1/2.

Joint with Chantal David and Ezra Waxman.

Olivier Ramaré (CNRS / U. Aix-Marseille)

Monday 8/7, 14:30 - 15:10.

The trigonometric polynomials on sums of two squares *et alia*

Let $\mathfrak{B}$ be the set of odd sums of two coprime integers and let $S(\alpha, N) = \sum_{\substack{b \le N \\ b \in \mathcal{B}}} e(b\alpha)$ be the associated trigonometric polynomial. We shall exploit a sieve identity to prove in particular that

$$S(\alpha, N) \ll N(\log N)^7 (q^{-1/2} + N^{-1/6} + (N/q)^{-1/2})$$

when $|q\alpha - a| \le 1/q$ and, for small q's, that

$$S(\alpha, N) \ll (N/\sqrt{\log N}) \biggl(\frac{\sqrt{q \log\log N} \log\log\log N}{\varphi(q)} + (\log N)^{-100} \biggr)$$

provided that $\log q \le (\log N)/(\log\log N)^2$ and $|\alpha - (a/q)| \le N^{-4/5}$. These are consequences of more general results. We recover in passing a form of the Harman sieve. As an application, we will study the number of representations of an integer as the sum of three primitive Gaussian integers, two of them being in a subset of $\mathfrak{B}$ of positive relative density.

This is joint work with Kasi Viswanadham.

Carlo Sanna (Politecnico di Torino)

Monday 8/7, 15:20 - 16:00.

On the number of residues of certain second-order linear recurrences

For every monic polynomial $f \in \mathbb{Z}[X]$ with $\deg(f) \geq 1$, let $\mathcal{L}(f)$ be the set of all linear recurrences with values in $\mathbb{Z}$ and characteristic polynomial f, and let

$$\mathcal{R}(f) := \big\{\rho(\boldsymbol{x}; m) : \boldsymbol{x} \in \mathcal{L}(f),\, m \in \mathbb{Z}^+\big\},$$

where $\rho(\boldsymbol{x}; m)$ is the number of distinct residues of $\boldsymbol{x}$ modulo m.

Dubickas and Novikas, motivated by some problems on fractional parts of powers of Pisot numbers, proved that $\mathcal{R}(X^2 - X - 1) = \mathbb{Z}^+$.

We generalize this result by showing that $\mathcal{R}(X^2 - a_1 X - 1) = \mathbb{Z}^+$ for every nonzero integer a_1. As a corollary, we deduce that for all integers $a_1 \geq 1$ and $k \geq 4$ there exists $\xi \in \mathbb{R}$ such that the sequence of fractional parts $\big(\{\xi\alpha^n\}\big)_{n\geq 0}$, where $\alpha := \big(a_1 + \sqrt{a_1^2 + 4}\big)/2$, has exactly k limit points. Our proofs are constructive and employ some results on the existence of special primitive divisors of certain Lehmer sequences.

This is a joint work with Federico Accossato.

Gérald Tenenbaum (U. Lorraine)

Monday 8/7, 16:10 - 16:50.

Mean-value of the Erdős-Hooley Delta function and applications

Defined by Erdős in 1973 and put forward by Hooley in 1979, the Delta function measures the logarithmic concentration of the divisors of an integer. Its average and normal behaviours raise delicate questions and motivated a vast bibliography. Recent progress due to Ford-Green-Koukoulopoulos, Koukoulopoulos-Tao, and Ford-Koukoulopoulos-Tao, introduce new approaches, without however exhausting the subject. In a joint work with Régis de la Bretèche, we improved the upper and lower bounds for the average order. I propose to describe the main tools that have been developed to tackle this theory and to discuss some applications to Waring-type and Diophantiine approximation problems.

Bruno Martin (U. Littoral Côte d'Opale)

Tuesday 9/7, 9:30 - 10:10.

On the smallest denominator of an interval

In 1977, Kruyswijk and Meijer conjectured that the average value of the smallest denominator of a rational number belonging to the interval $](j-1)/N, j/N]$, where $j = 1, \ldots, N$, is asymptotically equivalent to $16\pi^{-2}\sqrt{N}$, when N tends to infinity. In collaboration with Michel Balazard, we settled this conjecture in 2023. A few months later, Marklov came up with a new proof and extended our result in several directions. In this talk we will present our proof which relies on Weil's bound for Kloosterman sums and on a continuous version of Kruyswijk and Meijer's assertion, proved by Chen and Haynes in 2022.

Michel Balazard (CNRS / U. Aix-Marseille)

Tuesday 9/7, 10:20 - 11:00.

Real analysis and the distribution of prime numbers

I will describe some elements of real analysis and number theory used in the works of Selberg (1949), Wirsing (1962-64) and Lu (1999) on the error term in the Prime Number Theorem. An effective version of Lu's result, due to Gozé (2024), will also be mentioned.

Thomas Stoll (U. Lorraine)

Tuesday 9/7, 11:30 - 12:10.

Hensel's lemma for continuous p-adic functions

Hensel's lemma for polynomial p-adic functions $f : \mathbb{Z}_p \to \mathbb{Z}_p$ allows to deduce the existence of solutions to $f(x) = 0$ starting from an approximate solution. In 2016, E. Y. Axelsson and A. Khrennikov extended Hensel's lemma to 1- and p^α-Lipschitz functions and asked for a generalization of their result to general continuous p-adic functions. In this talk we present this generalization, obtained in recent joint work with H. Kaneko.

Marco Cantarini (U. Perugia)

Tuesday 9/7, 12:10 - 13:00.

Additive problems, Laplace convolutions and distributions

In this talk, we will speak about a recent work, written in collaboration with Alessandro Gambini and Alessandro Zaccagnini, that shows the link between the convolution in the sense of Laplace of the summatory function of an arithmetic function with itself and a general weighted average of a function that counts the number of representations (counting function) in a general additive problem. We will prove that such Laplace convolution has, in some sense, a central role in every weighted average and it is strictly connected to the so-called Cesàro average of the counting function with a specific order. Then, we will show that the main formulas give a connection between some specific tools from the theory of distributions. We will also present some interesting examples and possible further investigations.

Ilaria Viglino (EPFL)

Tuesday 9/7, 14:30 - 15:10.

Moment estimates of module lattice points for effective lattice constructions

We examine the moments of the number of lattice points in a fixed ball of volume V for lattices in Euclidean space which are modules over the ring of integers of a number field K. In particular, we show that moments obtained for "lifts of codes" to $\mathcal{O}_K$-modules converge to the Rogers integral formula for the space of free $\mathcal{O}_K$-module lattices. This extends work of Rogers for $\mathbb{Z}$-lattices. Joint work with Maryna Viazovska, Nihar Gargava and Vlad Serban.

Marc Munsch (U. Jean Monnet)

Tuesday 9/7, 15:20 - 16:00.

Sign changes of character sums on short intervals

In this talk, we will discuss quantitative results about the number of sign changes in the partial sums of real characters. Our method allows us to locate these changes on a very short initial interval (which goes beyond the range in Vinogradov's conjecture for the least quadratic non-residue).

The flexibility of our method allows us to deduce similar results in the case of random multiplicative functions. Related results can be obtained concerning the location of real zeros of Fekete polynomials F_D (the polynomials whose coefficients are the values of the real character χ_D).

This is based on a joint work with Oleksiy Klurman and Youness Lamzouri.

Matteo Verzobio (IST Austria)

Tuesday 9/7, 16:10 - 16:50.

Sieve methods for strong divisibility sequences

In this talk, we will investigate strong divisibility sequences and produce lower and upper bounds for the density of integers in the sequence which only have (somewhat) large prime factors. We will focus in particular on the special cases of Fibonacci numbers and elliptic divisibility sequences.

Joël Rivat (U. Aix-Marseille)

Wednesday 10/7, 09:30 - 10:10.

On the digits of primes numbers

Cathy Swaenepoel (U. Paris Cité)

Wednesday 10/7, 10:20 - 11:00.

Reversible primes

The properties of the digits of prime numbers and various other sequences of integers have attracted great interest in recent years. For any positive integer k, we denote by $\overleftarrow{k}$ the *reverse* of k in base 2, defined by

$$\overleftarrow{k} = \sum_{j=0}^{n-1} \varepsilon_j \, 2^{n-1-j} \quad \text{where} \quad k = \sum_{j=0}^{n-1} \varepsilon_j \, 2^j$$

with $\varepsilon_j \in \{0,1\}$, $j \in \{0,\dots,n-1\}$, $\varepsilon_{n-1} = 1$. A natural question is to estimate the number of primes $p \in [2^{n-1}, 2^n)$ such that $\overleftarrow{p}$ is prime. We will present a result which provides an upper bound of the expected order of magnitude. Our method is based on a sieve argument and also allows us to obtain a strong lower bound for the number of integers k such that k and $\overleftarrow{k}$ have at most 8 prime factors (counted with multiplicity). We will also present an asymptotic formula for the number of integers $k \in [2^{n-1}, 2^n)$ such that k and $\overleftarrow{k}$ are squarefree.

This is a joint work with Cécile Dartyge, Bruno Martin, Joël Rivat and Igor Shparlinski.

Alessandro Fazzari (U. Montréal)

Wednesday 10/7, 11:30 - 12:10.

Weighted statistics of families of L-functions

We discuss classical statistics of families of L-functions, weighted by the central value of the corresponding L-function. In particular, we focus on the weighted one-level density for the non-trivial zeros of L-functions, blending together the theory of moments and that of low lying zeros. We look at the case of the Riemann zeta function with special attention, trying to get a better understanding of the interplay between zeros and large values of zeta.

Javier Pliego-Garcia (U. Genova)

Wednesday 10/7, 12:20 - 13:00.

On Vu's theorem in Waring's problem

Answering a question of Nathanson about thin basis, Vu showed in 2000 the existence, for all $k > 1$ and some $s = s(k)$, of subsequences $X_k \subset \mathbb{N}$ satisfying that for every sufficiently large natural number n, the number of solutions of $n = x_1^k + ... + x_s^k$ with $x_i \in X_k$, which we denote by $R_s(X_k, n)$, satisfies the relation $R_s(X_k, n) \approx \log(n)$. Soon after the previous paper was published, Wooley (2003) improved the constraint on the number of variables. In this talk we shall discuss new results concerning problems within this circle of ideas.