Giornate INDAM di Teoria dei Numeri

Genova, 18-19 Dicembre 2017

Abstracts

Francesco Battistoni: Low discriminants for number fields of degree 8 and signature $(2,3)$

Let K be a number field of degree n, with discriminant d_K, and let r_1 be the number of real embeddings of K and r_2 be the number of couples of complex embeddings, so that $n = r_1 + 2r_2$.
A classical problem asks to establish the minimum value for $|d_K|$ when K ranges in the set of fields with a fixed signature (r_1, r_2). During the last century many methods for answering the question were set: from the classical tools of Geometry of Numbers invented by Minkowski to the analytic estimates involving the Dedekind Zeta functions, due to Odlyzko [2], Poitou [5] and Serre [6] up to the algorithmic procedures, based on number-geometric ideas, developed by Pohst [3], Martinet [1] and Diaz y Diaz [4] (in collaboration with the previous authors): with these ideas the problem was solved for $n \leq 7$, with any signature, and also for $n = 8$, if the signature is either $(8,0)$ or $(0,4)$.
In this work we exploit the methods aforementioned in order to prove the following results:

Theorem 1 *Let d_K be the discriminant of a number field K with degree 8 and signature $(2,3)$. Then the minimum value of $|d_K|$ is equal to 4286875.*

Theorem 2 *There are 56 number fields of degree 8 and signature $(2,3)$ with $|d_K| \leq 5726300$; with the exception of two non-isomorphic fields with $|d_K| = 5365963$, every field in the list is uniquely characterized by the value of $|d_K|$.*

References

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[2] A. M. Odlyzko, *Bounds for discriminants and related estimates for class numbers, regulators and zeros of zeta functions: a survey of recent results*, Sém. Théor. Nombres Bordeaux (2) **2** (1990), no. 1, 119–141.

[3] M. Pohst, *On the computation of number fields of small discriminants including the minimum discriminants of sixth degree fields*, J. Number Theory **14** (1982), no. 1, 99–117.

[4] M. Pohst, J. Martinet, and F. Diaz y Diaz, *The minimum discriminant of totally real octic fields*, J. Number Theory **36** (1990), no. 2, 145–159.

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Sandro Bettin: Statistical distribution of the Stern sequence

This is joint work with Sary Drappeau and Lukas Spiegelhofer [1].

The Stern sequence $s(n)$ is defined recursively as follows:

$$s(1) = 1, \qquad s(2n) = s(n), \qquad s(2n+1) = s(n) + s(n+1),\ \forall n \geq 1.$$

We show that the values $\log s(n)$, with n taken uniformly randomly in $I_n := [2^{N-1}, 2^N)$, are asymptotically distributed according to a normal law as $N \to \infty$.

This is obtained by expressing the $s(n)$ with $n \in I_n$ as the denominators of the N-th preimages at $0, 1$ of the Farey map and then studying the relevant transfer operator.

References

[1] S. Bettin, S. Drappeau and L. Spiegelhofer, *Statistical distribution of the Stern sequence*, arXiv:1704.05253

Dante Bonolis: On the size of the maximum of incomplete Kloosterman sums

For any $t : \mathbb{F}_p \to \mathbb{C}$, one can define

$$M(t) := \frac{1}{\sqrt{p}} \max_{0 \le H < p} \left| \sum_{n<H} t(n) \right|.$$

The Pólya-Vinogradov inequality implies that

$$M(t) \ll \|K\|_\infty \log p.$$

Where K is the Fourier transform of t. Our goal is to understand if this bound is sharp for example when

$$t : x \mapsto e\Big(\frac{ax + b\overline{x}}{p}\Big)$$

where $e(\cdot) := \exp(2\pi i \cdot)$, $a, b \in \mathbb{F}_p^\times$ and $\overline{x}$ denotes the inverse of x modulo p (i.e. for incomplete Kloosterman sums). An other question we will try to answer is: fix $A > 0$, how many $a \in \mathbb{F}_p^\times$ there are such that $M(e(\frac{ax+\overline{x}}{p})) > A$?

Giacomo Cherubini: The prime geodesic theorem for the Picard manifold

I will recall the prime geodesic theorem in two dimensions and extend the construction to three dimensions. The main result is a new bound for the error term in the problem, which improves on the currently best known unconditional estimate, due to Sarnak. This is joint work with D. Chatzakos and N. Laaksonen.

Giovanni Coppola: A new elementary property of correlations

Starting from the recent arxiv paper (of same title), we give a brief panorama of the new method involving finite Ramanujan expansions, regarding its applications, to shifted convolution sums, also called correlations, of any arithmetic functions f, g. If time permits, we'll see how these, say, "single correlations", provide new formulae, for averages of correlations; in particular, for the Selberg integral of f and g (a mean-square of short intervals f & g values on a long interval), in case f & g satisfy Ramanujan Conjecture.

Giuseppe Molteni: An effective Chebotarev theorem under GRH

Given a Galois extension of fields $\mathbb{K}/\mathbb{L}$, Chebotarev's theorem shows that the image of the Frobenius map of prime ideals in $\mathbb{K}$ are equidistributed among the conjugation classes of the Galois group of the extension.
We show a version of this theorem which is completely explicit, i.e. where the remainder term is explicitly given in terms of the degree and the discriminant of $\mathbb{L}$.
The result has been obtained in collaboration with L. Grenié [1] and improves an analogous result of J. Oesterlé [2].

References

[1] L. Grenié, and G. Molteni, *An effective Chebotarev density theorem under GRH*, Preprint http://arxiv.org/abs/1709.07609, 2017.

[2] J. Oesterlé, *Versions effectives du théorème de Chebotarev sous l'hypothèse de Riemann généralisée*, Astérisque **61** (1979), 165–167.

Nadir Murru: Convergence and Periodicity of Multidimensional Continued Fractions

Multidimensional continued fractions generalize classical continued fractions with the aim of providing periodic representations of algebraic irrationalities by means of integer sequences. We see some results regrading their convergence and periodicity focusing on the Jacobi-Perron algorithm. In particular, we see that partial quotients of a multidimensional continued fraction are periodic if and only if numerators and denominators of convergents are linear recurrence sequences, generalizing similar results that hold for classical continued fractions.

References

[1] L. Bernstein, *New infinite classes of periodic Jacobi–Perron algorithms*, Pacific Journal of Mathematics, Vol. **16**, No. **3**, 439–469, (1965).

[2] L. Bernstein, *The Jacobi–Perron algorithm – its theory and application*, Lectures Notes in Mathematics, Vol. **207**, 1971.

[3] N. Murru, *On the periodic writing of cubic irrationals and a generalization of Rédei functions*, International Journal of Number Theory, Vol. **11**, No. **3**, 779–799, 2015.

[4] N. Murru, *Linear recurrence sequences and periodicity of multidimensional continued fractions*, The Ramanujan Journal, Vol. **44**, No. **1**, 115–124, 2017.

Giovanni Panti: Continued fractions on triangle groups

We identify a continued fraction algorithm with the Gauss-type map it induces on an interval in $\mathbb{P}^1(\mathbb{R})$. We will show that in the classical case (i.e., working over the integers) the subgroups of $PGL_2(\mathbb{R})$ generated by the branches of these maps have index at most 8 in the extended modular group $PGL_2(\mathbb{Z})$. This has an impact on the validity of the Serret theorem (two reals are $PGL_2(\mathbb{Z})$-conjugated iff they have the same cf tail) for these algorithms, and we will settle the issue.

As a natural generalization, one might replace the extended modular group with other hyperbolic triangle groups; for example, cf algorithms based on the Hecke groups $(2, n, \infty)$ appear immediately when dealing with the geodesic flow on translation surfaces. Here the situation is much more involved, and I will discuss a few results, techniques, and open problems.

Francesco Pappalardi: Punti mai primitivi per curve ellittiche

In analogia con la classica congettura di Artin per radici primitive, nel 1977, S. Lang e H. Trotter hanno congetturato che, data una curva ellittica $E/\mathbb{Q}$ e un punto $P \in E(\mathbb{Q})$ di ordine infinito, l'insieme dei primi di buona riduzione p per cui $\langle P \bmod p\rangle = E(\mathbb{F}_p)$, ha una densità $\delta_{E,P}$. Nel seminario ci occupiamo della classificazione delle curve e dei punti per cui $\delta_{E,P} = 0$ utilizzando l'azione del gruppo di Galois $\mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$ sull'insieme $\frac{1}{\ell}P = \{Q \in E(\overline{\mathbb{Q}}) : \ell Q = P\}$ ed ℓ è primo.

Alberto Perelli: Formule esplicite per medie di rappresentazioni di Goldbach

Presentiamo una formula esplicita, analoga alla classica formula esplicita per $\psi(x)$, per le medie di Cesàro-Riesz di ogni ordine $k > 0$ del numero di rappresentazioni di n come somma di due primi. Il metodo si basa su una trasformata di Mellin doppia e sul prolungamento analitico di certe funzioni ad essa collegate (lavoro in collaborazione con J.Brüdern e J.Kaczorowski).

Mattia Righetti: Zeros of linear twists of L-functions

In this talk we will give an exposition of the results on the existence of zeros of linear twists of L-functions, and in particular of the Hurwitz-Lerch zeta functions, for $\sigma > 1$. If time permits we will also discuss the situation for $1/2 < \sigma < 1$.

Carlo Sanna: A coprimality condition on consecutive values of polynomials

This is joint work with Márton Szikszai.

Given a sequence of integers $s = (s(n))_{n\geq 1}$, let $G_s \geq 2$ be the smallest integer such that for every integer $k \geq G_s$ one can find k consecutive terms of s with the property that none of them is coprime to all the others. Of course, G_s may not exist.

Erdős [2] proved the existence of G_s when s is the sequence of natural numbers, and in such a case the combined efforts of Pillai [3, 4] and Brauer [5] showed that $G_s = 17$. Later, Evans [6] proved the existence of G_s when s is an arithmetic progression.

The study of G_s has at least two motivations: for Erdős and Brauer it came from a problem on prime gaps, while for Pillai it came from the classical Diophantine problem whether the product of consecutive integers can be a perfect power.

We prove the existence of G_s in the case in which $s = (f(n))_{n\geq 1}$, where $f \in \mathbb{Z}[X]$ is a quadratic of cubic polynomial. This answers a question of Harrington and Jones [7]. Our proof relies on results on the p-adic valuations of products of consecutive polynomial values, on elementary properties of the roots of f modulo a prime, and on lower bounds for the number of certain primes dividing the values of an auxiliary polynomial.

References

[1] C. Sanna and M. Szikszai, *A coprimality condition on consecutive values of polynomials*, Bull. Lond. Math. Soc. **49** (2017), 908–915.

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[5] A. Brauer, *On a property of k consecutive integers*, Bull. Amer. Math. Soc. **47** (1941), 328–331.

[6] R. Evans, *On N consecutive integers in an arithmetic progression*, Acta Sci. Math. (Szeged) **33** (1972), 295–296.

[7] J. Harrington and L. Jones, *Extending a theorem of Pillai to quadratic sequences*, Integers **15A** (2015), Paper No. A7, 22.

Alessandro Zaccagnini: Additive problems with prime variables

We give a survey of recent results, obtained in collaboration with Alessandro Languasco, of additive problems with prime variables. In particular, we will talk of weighted averages of the number of representation of an even integer as a sum of two primes, and related problems.

Giamila Zaghloul: On the linear twist of degree-1 functions in the extended Selberg class

Let F be a degree-1 function in the extended Selberg class $\boldsymbol{S}^\sharp$ and $\alpha \in \mathbb{R}$. We study the main analytic properties of the linear twist $F(s, \alpha)$. Starting from the characterization of $\boldsymbol{S}_1^\sharp$ and the known properties of the Hurwitz-Lerch zeta function, we show that the linear twist satisfies a functional equation of Hurwitz-Lerch type. We also discuss some results on the polynomial growth and the distribution of trivial zeros.